Mechanica en Relativiteit 1

toets Relativiteitstheorie 13-12-2010

1 (a) ① Snelheden kunnen niet groter zijn dan de lichtsnelheid.

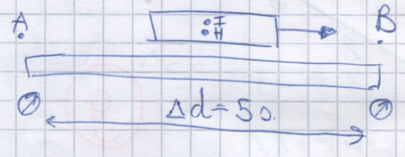
0 ② Ook tijd is relatief en stelselafhankelijk $t \neq t'$

9,33

(b)

	coordinaattijd	eigentijd	ruimtetijd
Henk	JA	Nee	Nee
Ingrid	JA	JA	JA

2



$\Delta t = 7\,s$

(c) $\Delta S^2 = \Delta t^2 - \Delta d^2 = 7^2 - 5^2 = 49 - 25 = 24\,s^2$

1½ $\Delta S = \sqrt{\Delta S^2} = \sqrt{24} = \sqrt{4}\cdot\sqrt{6} = 2\sqrt{6}$ sec.

1½ (d) $\Delta S^2 > 0 \longrightarrow$ tijdachtig

(e) Ja ; wanneer het ruimte-tijdinterval tijdachtig is bestaat er een inertiaalstelsel waarin event A en B op dezelfde plaats plaatsvinden.

2 $\Delta x' = 0$ $\quad A(0,0)$, $B(5,7)$ $\Big\}$ (d, t) - coördinaten

In het 2-waarnemersdiagram, moet de t'-as door deze punten lopen:

helling t'-as $= \frac{1}{\beta} = \frac{\Delta t}{\Delta x} = \frac{7}{5} \longrightarrow \beta = \frac{5}{7}$

De snelheid t.o.v. het Home Frame is $\frac{5}{7}$ (van de lichtsnelheid.)

7

2

$t = 0$

α $\longrightarrow v_\alpha = 0{,}2 = \frac{1}{5}$

$\longrightarrow v_\beta = 0{,}6 = \frac{3}{5}$

β

$t = 5h$

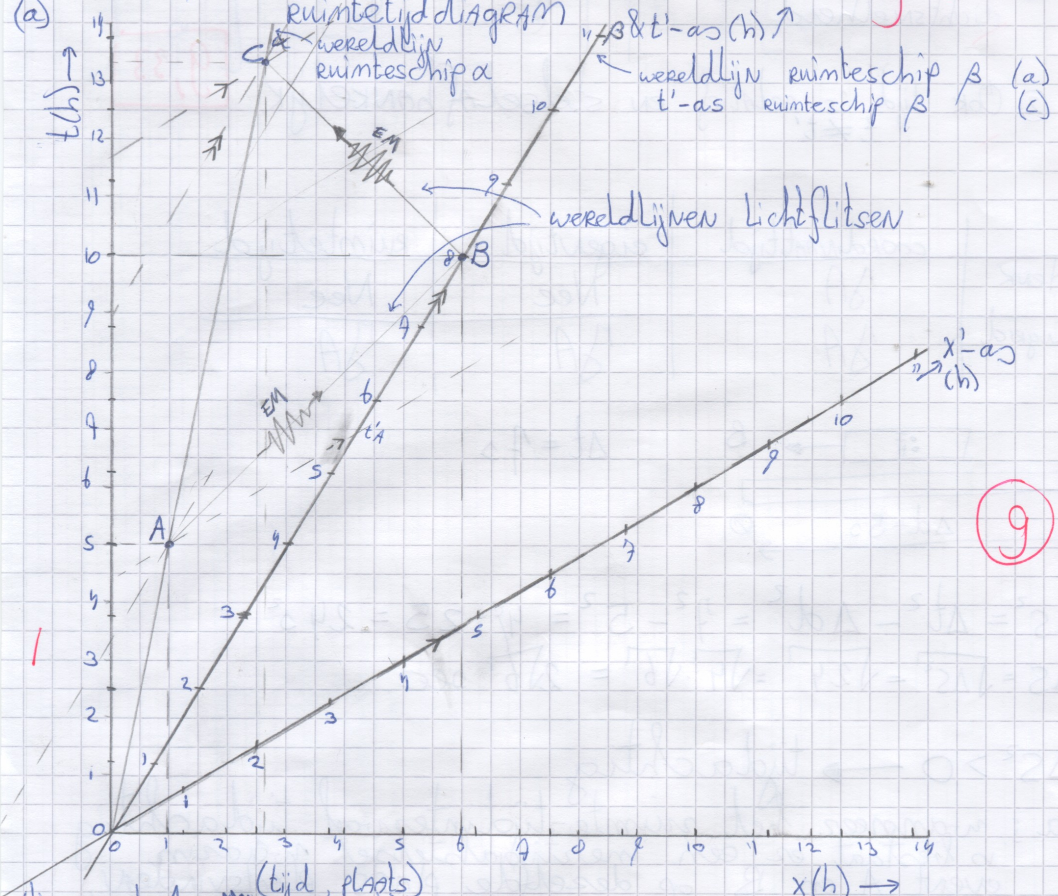
$\alpha \longrightarrow v_\alpha = 0{,}2$

licht

licht $\beta \longrightarrow v_\beta = 0{,}6$

$\boxed{9{,}33}$

(a)



9

1

(b) (tijd, plaats)

event A: (5h, 1h)

$1\frac{1}{2}$ event B: (10h, 6h)

event C: (13,3h; 2,6h)

(c) helling t'-as $= \frac{1}{\beta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$

$1\frac{1}{2}$ Hier geldt $\Delta x' = 0 \longrightarrow \Delta s^2 = \Delta t^2 - \Delta x^2 = \Delta t'^2 - \Delta x'^2$

$\Delta t^2 - (\beta \Delta t)^2 = \Delta t'^2$

$(1 - \beta^2)\Delta t^2 = \Delta t'^2$

$\Delta x = \gamma \cdot \Delta x'$ ← zo ook $\Delta t = \gamma \cdot \Delta t'$ met $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

schaalverdeling

helling x'-as $= \frac{1}{\text{helling t'-as}} = \frac{1}{\frac{5}{3}} = \frac{3}{5} = \beta$

$\gamma = \frac{1}{\sqrt{1 - \frac{3}{5}^2}} = \frac{1}{\sqrt{1 - \frac{9}{25}}} = \frac{1}{\sqrt{\frac{16}{25}}} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

(tijd, plaats) in Other Frame.

event A: $(5{,}5h ; -2{,}5h^*)$

event B: $(8h ; 0h)$

event C: $(-\cancel{8}14{,}1h^* ; -6\tfrac{3}{4}h^*)$

* bepaling x'_A:

$t_A = \frac{5}{3} x_A + b$ door $x=1$ en $t=5$

$5 = \frac{5}{3} \cdot 1 + b$

$b = 3\frac{1}{3}$

$t_A = \frac{5}{3} x_A + 3\frac{1}{3} = \frac{3}{5} x_A$

$\frac{25}{15} x_A + \frac{10}{3} = \frac{9}{15} x_A$

$\frac{16}{15} x_A = -\frac{10}{3}$

$x_A = \frac{-150}{16 \cdot 3} = -3\frac{1}{8}$

$\Delta x_A = \gamma \cdot \Delta x'_A$

$\Delta x'_A = \sqrt{1-\beta^2} \cdot \Delta x_A$

$\Delta x'_A = \frac{4}{5} \cdot -3\frac{1}{8} = -2{,}5h.$

* bepaling x'_c:

$t_c = \frac{5}{3} x_c + b$ door $x = 2{,}6$ en $t = 13{,}3$

$13{,}3 = \frac{5}{3} \cdot 2{,}6 + b \rightarrow b \approx 9{,}0$

$t_c = \frac{5}{3} x_c + 9 = \frac{3}{5} x_c$

$\frac{25}{15} x_c + 9 = \frac{9}{15} x_c$

$\frac{16}{15} x_c = -9$

$x_c = -8\,9/16$

$x_c = \gamma\, x'_c$

$x'_c = \sqrt{1-\beta^2}\; x_c$

$x'_c = \frac{4}{5} \cdot -8\,9/16 = -6\frac{3}{4}h.$

* bepaling t'_c:

$t_c = \frac{3}{5} x_c + b$ door $x = 2{,}6$ en $t = 13{,}3$

$13{,}3 = \frac{3}{5} \cdot 2{,}6 + b$

$b = 11{,}74$

$t_c = \frac{3}{5} x_c + 11{,}74 = \frac{5}{3} x_c$

$\frac{9}{15} x_c - \frac{25}{15} x_c = -11{,}74 = -\frac{16}{15} x_c$

$x_c = 11{,}00625$

$t_c^* = \frac{3}{5} x_c + 11{,}74$

$t_c = \frac{3}{5} \times 11 + 11{,}74 = 17{,}60375h.$

$t'_c = \sqrt{1-\beta^2} \cdot t_c$

$t'_c = \frac{4}{5} \cdot 17{,}6 = 14{,}083h.$

e $x = \gamma(x' + \beta \cdot t')$ $\qquad t = \gamma(t' + \beta x')$

$x' = \gamma(x - \beta \cdot t)$ $\qquad t' = \gamma(t - \beta x)$.
(tijd, plaats)

A: $\left.\begin{array}{l} x'_A = \frac{5}{4}\left(1 - \frac{3}{5}\cdot 5\right) = -2{,}5\,h \\ t'_A = \frac{5}{4}\left(5 - \frac{3}{5}\cdot 1\right) = 5{,}5\,h \end{array}\right\} A(5{,}5h; -2{,}5h)$

B: $\left.\begin{array}{l} x'_B = \frac{5}{4}\left(6 - \frac{3}{5}\cdot 10\right) = 0\,h \\ t'_B = \frac{5}{4}\left(10 - \frac{3}{5}\cdot 6\right) = 8\,h \end{array}\right\} B(8h; 0h)$

2

C: $\left.\begin{array}{l} x'_C = \frac{5}{4}\left(2{,}6 - \frac{3}{5}\cdot 13{,}3\right) = -6{,}725\,h \\ t'_C = \frac{5}{4}\left(13{,}3 - \frac{3}{5}\cdot 2{,}6\right) = 14{,}675\,h \end{array}\right\} C(14{,}675h; -6{,}725h)$

f. $v'_x = \frac{\Delta x'}{\Delta t'} = \frac{\gamma(\Delta x - \beta \Delta t)}{\gamma(\Delta t - \beta \Delta x)} \frac{\frac{1}{\Delta t}}{\frac{1}{\Delta t}} = \frac{v_x - \beta}{1 - \beta v_x}$

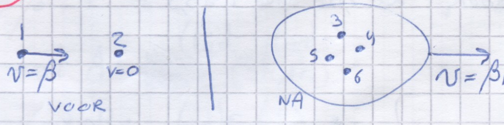
2 $\left.\begin{array}{l} \beta = 0{,}6 \\ v_x = 0{,}2 \end{array}\right\} v'_x = \frac{0{,}2 - 0{,}6}{1 - 0{,}6 \times 0{,}2} = \frac{-0{,}4}{\frac{22}{25}} = -\frac{2}{5}\cdot\frac{25}{22} = -\frac{50}{110} = -\frac{5}{11}$

$v'_x = -\frac{5}{11}$ $\qquad \left(= \frac{-2.5}{5.5}\right)$

3 (a) 9

$$\overset{1}{\begin{bmatrix} E_1 \\ p_{1x} \\ p_{1y} \\ p_{1z} \end{bmatrix}} + \overset{2}{\begin{bmatrix} E_2 \\ p_{2x} \\ p_{2y} \\ p_{2z} \end{bmatrix}} = \overset{3}{\begin{bmatrix} E_3 \\ p_{3x} \\ p_{3y} \\ p_{3z} \end{bmatrix}} + \overset{4}{\begin{bmatrix} E_4 \\ p_{4x} \\ p_{4y} \\ p_{4z} \end{bmatrix}} + \overset{5}{\begin{bmatrix} E_5 \\ p_{5x} \\ p_{5y} \\ p_{5z} \end{bmatrix}} + \overset{6}{\begin{bmatrix} E_6 \\ p_{5x} \\ p_{5y} \\ p_{5z} \end{bmatrix}}$$

met $E = \gamma \cdot m$
met $p_x = \gamma \cdot m \cdot \beta$
met $p_y = p_z = 0$
als beweging in x-richting



$1 \to p$
$2 \to p$
$3 \to p$
$4 \to p$
$5 \to K^-$
$6 \to K^+$

$\boxed{\text{zie laatste pagina}}$

$\boxed{\text{Totaal: kijk naar } 1+2}$

(b) $p_t^2 - p_x^2 - p_y^2 - p_z^2 = \left(m \cdot \frac{dt}{d\tau}\right)^2 - \left(m \cdot \frac{dx}{d\tau}\right)^2 - \left(m \cdot \frac{dy}{d\tau}\right)^2 - \left(m \cdot \frac{dz}{d\tau}\right)^2$

$= (\gamma m + m)^2 - (\gamma m \cdot \beta + 0)^2 - 0 - 0$

3

$= \gamma^2 m^2 + 2\gamma m^2 + m^2 - \gamma^2 m^2 \beta^2 \overset{?}{=} (2\gamma + 2) m^2 = 2\gamma m^2 + 2m^2$

$= \gamma^2 m^2 - \gamma^2 m^2 \beta^2 + m^2 + 2\gamma m^2$ $\qquad E_1^2 - p_{1x}^2 = m^2$

$= E_1^2 - p_{1x}^2 + m^2 + 2\gamma m^2$

$= m^2 + m^2 + 2\gamma m^2 = 2\gamma m^2 + 2m^2$ $\qquad$ QED.

Hier: $\boxed{\gamma = \gamma_1}$ $\qquad \boxed{m = m_p}$

γ $\boxed{202\ 3a}$

$m_p = 938{,}3\,MeV$

$m_K = 493{,}6\,MeV$

$E_T^2 - p_T^2 = (2\gamma + 2)m^2$

$E_T = E_{p_1} + E_{p_2}$

$E_T = \gamma \cdot m + m = m(1+\gamma)$

Behoud VAN 4-impuls:

3

$m_p(1+\gamma_1) = \gamma_2 m_p + \gamma_2 m_p + \gamma_2 m_K + \gamma_2 m_K$

(1) $m_p(1+\gamma_1) = 2\gamma_2 m_p + 2\gamma_2 m_K = \underbrace{2\gamma_2(m_p + m_K)}_{E_T}$ (E)

(2) $\gamma_1 m_p \cdot \beta = 2(\gamma_2 \cdot \beta_1 m_p) + 2(\gamma_2 \cdot \beta_1 m_K) = \underbrace{2\gamma_2 \beta_1 (m_p + m_K)}_{P_T}$ (P)

$$\boxed{\begin{aligned} K_p &= E_p - m_p \\ K_p &= \gamma_1 m_p - m_p = (\gamma_1 - 1)\cdot m_p \end{aligned}}$$

$\gamma_1 = \frac{1}{\sqrt{1-\beta^2}}$

$\gamma_2 = \frac{1}{\sqrt{1-\beta_1^2}}$

~~(1) $m_p(1 + \frac{1}{\sqrt{1-\beta^2}}) = 2\frac{1}{\sqrt{1-\beta_1^2}} m_p + 2\frac{1}{\sqrt{1-\beta_1^2}}$~~

(1) $1 + \gamma_1 = \frac{2\gamma_2(m_p + m_K)}{m_p}$

$\gamma_1 = \frac{2\gamma_2(m_p + m_K)}{m_p} - 1$

$K_p = 2\gamma_2(m_p + m_K) - 2m_p$

$\gamma_2 = ?$

$(2\gamma_1 + 2)m_1^2 = \overbrace{(2\gamma_2(m_p+m_K))^2}^{E_T^2} - \overbrace{(2\gamma_2\beta_1(m_p+m_K))^2}^{P_T^2}$

$= 4\gamma_2^2(m_p+m_K)^2 - 4\gamma_2^2\beta_1^2(m_p+m_K)^2$

$= 4\gamma_2^2(m_p+m_K)^2(1-\beta_1^2)$

$= 4 \cdot \cancel{\frac{1}{1-\beta_1^2}}(m_p+m_K)^2 \cdot \cancel{(1-\beta_1^2)}$

$= 4(m_p+m_K)^2$

behouden "grootheid"

$\boxed{K_p \approx 2494\,MeV}$

3

$2\gamma_1 + 2 = \frac{4(m_p+m_K)^2}{m_1^2}$

$\gamma_1 = \frac{4(m_p+m_K)^2}{2m_1^2} - 1 = \frac{2(m_p+m_K)^2}{m_1^2} - 1$

$\gamma_1 = \frac{2(938{,}3 + 493{,}6)^2}{938{,}3^2} - 1 \approx 3{,}66$

$K_p = (3{,}66 - 1)\cdot 938{,}3\,MeV$

$K_p = 2493{,}7\,MeV$

3 a

$$\overset{1}{\begin{bmatrix} \gamma_1 m_p \\ \gamma_1 m_p \beta \\ 0 \\ 0 \end{bmatrix}} + \overset{2}{\begin{bmatrix} m_p \\ 0 \\ 0 \\ 0 \end{bmatrix}} = \overset{3}{\begin{bmatrix} m_p \gamma_2 \\ m_p \gamma_2 \cdot \beta_1 \\ 0 \\ 0 \end{bmatrix}} + \overset{4}{\begin{bmatrix} m_p \gamma_2 \\ m_p \gamma_2 \beta_1 \\ 0 \\ 0 \end{bmatrix}} + \overset{5}{\begin{bmatrix} m_K \gamma_2 \\ m_K \cdot \gamma_2 \cdot \beta_1 \\ 0 \\ 0 \end{bmatrix}} + \overset{6}{\begin{bmatrix} m_K \cdot \gamma_2 \\ m_K \cdot \gamma_2 \cdot \beta_1 \\ 0 \\ 0 \end{bmatrix}}$$